CONVECTIVE STABILITY OF A HORIZONTAL LAYER OF CHEMICALLY ACTIVE LIQUID IN THE PRESENCE OF TRANSVERSE FLOW OF REACTANT

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This work investigates conditions of the appearance of convection in a horizontal layer of a chemically active liquid with penetrable boundaries held at the same temperature. Heat is liberated throughout the volume of the liquid as a result of a zero-order reaction. There is uniform transverse movement of reactant through the penetrable boundaries at a constant rate. The possibility of the existence of time-independent processes of heat transfer in such a system, with free convection absent, was analyzed in [1]. The linear problem of convective stability under stable heat-conduction conditions, described in [1], is solved here. We obtained, for different values of the parameters, limits of stability defining the threshold of the beginning of convection. The convective stability of reacting liquids in the absence of sparging was examined in [2-4]. The effect of transverse movement on the convective stability of a horizontal layer of nonreactive liquid was studied in [5].

An infinite horizontal layer of a incompressible, chemically active liquid is bounded by solid permeable surfaces $z = 0$ and $z = d$. A homogeneous exothermic reaction with a high rate of heat production is occurring in the liquid. The rate of heat liberation permits us to describe the process through a model of a zero-order reaction. In accordance with Arrhenius' law, the dependence of the capacity of the heat surfaces on temperature is described by the function $k_0 \exp[-E/(RT)]$, where T is absolute temperature, R is the gas constant, preexponential multiplier k_0 and activation energy E are parameters of the reaction. The boundaries of the layer are kept at constant temperature T_0. The reactant is uniformly injected through the bottom boundary at a rate v_0, while it is uniformly drawn out through the top boundary at that same rate. This leads to the existence in the layer of unperturbed lateral flow with a uniform vertical speed v_0.

As follows from [1], stable heat-transfer processes are possible under conditions of nonconvective heat exchange in a reactant undergoing uniform transverse movement. The corresponding temperature distributions are described by solutions to a one-dimensional nonlinear equation of thermal conductivity appearing as follows in dimensionless form:

$$\Theta_0'' - \mathrm{Pe}\,\Theta_0' + \delta \exp[\Theta_0/(1+\beta_0\Theta_0)] = 0, \qquad (1)$$

with boundary conditions $\Theta_0(0) = \Theta_0(1) = 0$. Here $\Theta_0(z)$ is the unperturbed value of the temperature, calculated from the temperature of the boundaries of the layer; the units of temperature measurement here are RT_0^2/E; $\mathrm{Pe} = v_0 d/\chi$ is the Péclet number characterizing the rate of injection; $\delta = Qk_0Ed^2 \exp[-E/(RT_0)]/(\varkappa RT_0^2)$ is the Frank-Kamenetskii parameter; $\beta_0 = RT_0/E$ is a small parameter whose value for combustion reactions does not exceed 0.05 and, further it is usually [4, 6] assumed that $\beta_0 = 0$; Q is the calorific value of the reaction; χ and $\varkappa$ are the coefficients of thermal diffusivity and thermal conductivity, respectively.

In the special case when $\mathrm{Pe} = 0$ (absence of injection), Eq. (1) reduces to the well-known equation of steady-state thermal-explosion theory, the solution of which was described in [6]. According to [6], steady-state states of the system exist only within the range $0 \le \delta \le \delta_{cr}$. The upper boundary of this interval $\delta_{cr} = 3.514$ defines the threshold of thermal explosion. Two equilibrium modes of heat conduction are possible within the range $\delta < \delta_{cr}$. The corresponding stationary temperature distributions are symmetric relative to the middle of the layer, where the temperature is at a maximum.

In the general case ($\mathrm{Pe} \ne 0$), the solution to the boundary-value problem (1) shows that there are also (as when $\mathrm{Pe} = 0$) two stationary modes of heat transfer for the case of a fixed Péclet number within the range of values of δ in the reactant bounded from above by δ_{cr}. As before, the value of δ_{cr} characterizes the thermal-explosion threshold and increases with increase in Pe. The function $\delta_{cr}(\mathrm{Pe})$ at $\mathrm{Pe} < 4$ is closely approximated by the polynomial

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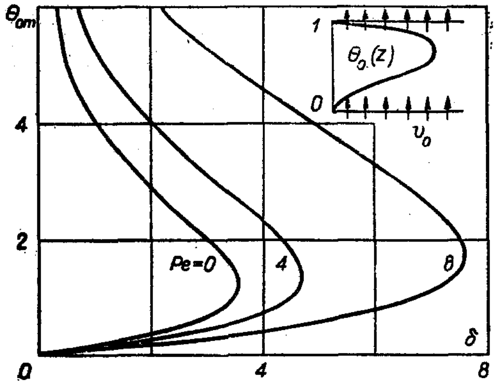

Fig. 1

$$\delta_{cr} = 3.514 + 0.0485\mathrm{Pe}^2 + 0.0015\mathrm{Pe}^4.$$

For both solutions, lateral flow destroys the symmetry of the temperature profiles and the zone of greatest heating is shifted to the boundary z = 1. Figure 1 shows the dependence of the maximum temperature in the layer Θ_{0m} on the Frank-Kamenetskii parameter δ for several Péclet numbers (Pe = 0, 4, 8), along with a schematic representation of an unperturbed temperature distribution typical for the case of injection. At any values of Pe, the upper stationary solution in the heat-conduction mode proves to be unstable.

Due to convective instability, the lower stationary heat-conduction mode in a moving reactant may also be disrupted (the stratification of density in the top part of the layer is unstable). It is the convective stability of this mode that is of the greatest interest, and it will be the subject of further study here.

The convection equation in the Boussinesq approximation for a homogeneous chemically active liquid [2-4] differs from the usual equations [7] by the inclusion of a heat-conductivity term describing an internal heat liberation that increases exponentially with temperature. To determine the conditions of the beginning of convection, we will examine the behavior of small perturbations for a rate **v**, temperature Θ, and pressure p. The dimensionless linearized perturbation equations have the form

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{\mathrm{Pe}}{\mathrm{Pr}}(\gamma\nabla)\mathbf{v} = -\nabla p + \Delta\mathbf{v} + \mathrm{Ra}\,\Theta\gamma,$$
$$\mathrm{Pr}\frac{\partial\Theta}{\partial t} + \mathrm{Pe}\,\gamma\nabla\Theta + \mathbf{v}\Delta\Theta_0 = \Delta\Theta + \delta\Theta\exp\Theta_0,\ \mathrm{div}\,\mathbf{v} = 0, \tag{2}$$

where t is the time; γ is a unit vector directed upward along the z axis; $\mathrm{Pr} = \nu/\chi$ is the Prandtl number; $\mathrm{Ra} = g\beta RT_0^2 d^3/E\nu\chi$ is the Rayleigh number; g is the acceleration due to gravity; β and ν are the coefficients of volume expansion and kinematic viscosity of the liquid, respectively. As units of measurement of distance and temperature, as in the stationary problem (1), we have chosen the values d and RT_0^2/E; for time, rate, and pressure, we have selected d^2/ν, χ/d, and $\rho_0\chi\nu/d^2$.

At the boundaries of the layer, the rate and temperature perturbations vanish:

$$\mathbf{v} = 0,\ \Theta = 0 \ \text{at} \ z = 0 \ \text{and} \ z = 1. \tag{3}$$

Let us introduce normal perturbations dependent on time and the horizontal coordinates according to the law $\exp[-\lambda t + i(k_1 x + k_2 y)]$, where k_1 and k_2 are real wave numbers; $\lambda = \lambda_r + i\lambda_i$ (λ_r is the real portion of the decrement λ, and λ_i is the imaginary portion of the same). After excluding the pressure from (2), the boundary-value problem for the amplitude of normal perturbations of rate w(z) and temperature θ(z) acquires the form

$$-\lambda(w'' - k^2 w) + \frac{\mathrm{Pe}}{\mathrm{Pr}}(w''' - k^2 w') = (w^{\mathrm{IV}} - 2k^2 w'' + k^4 w) - \mathrm{Ra}\,k^2\theta, \tag{4}$$

$$-\lambda\,\mathrm{Pr}\,\theta + \mathrm{Pe}\,\theta' + w\Theta_0' = (\theta'' - k^2\theta) + \delta\theta\exp\Theta_0\ (k^2 = k_1^2 + k_2^2); \tag{5}$$

$$w = 0,\ w' = 0,\ \theta = 0 \ \text{at} \ z = 0 \ \text{and} \ z = 1.$$

The temperature distributions $\Phi_0(\mathrm{Pe}, \delta, z)$ are the solutions of (5), the unperturbed stationary problem (1) and exist within the range $\delta \leq \delta_{cr}$.

At Ra = 0 (absence of bouyancy), boundary-value problem (4), (5) reduces to the problem of the stability of nonconvective heat-transfer processes in a laterally moving reactant relative to temperature perturbations [1]. In the case Pe = 0, the system (4), with boundary conditions (5), coincides with the system examined in [2-4].

The result of the solution of convective stability problem (4), (5) is the determination of the decrements of normal perturbations λ in relation to five dimensionless parameters: the Prandtl, Péclet, and Rayleigh numbers, the Frank-Kamenetskii parameter, and the wave number. In finding the eigenvalues $\lambda(Pr, Pe, Ra, \delta, k)$, Eq. (4) is represented in the form of a system of twelve ordinary first-order differential equations for the real and imaginary parts of the complex amplitudes of the perturbations and their derivatives. We plotted numerically three linearly independent particular solutions for which the conditions were satisfied at the initial point of integration. We used the Runge–Kutta method to construct the particular solutions. (The use of this method to solve problems of convective stability is discussed in [8]). The critical conditions for the occurrence of convection are the values of the parameters in which $\lambda_r = 0$; states with $\lambda_r > 0$ are stable, while states in which $\lambda_r < 0$ are unstable.

The calculations show that the spectrum of decrements $\lambda(Ra)$ has a very complicated structure, similar to that in [4] for the case Pe = 0. In particular, in the region of stability ($\lambda_r > 0$), oscillatory modes with $\lambda_i \neq 0$ are possible. For convective instability, however, the real branches of the spectrum are important, and the critical perturbations are monotonic. The presence of injection leads to a situation whereby the boundary of monotonic instability ($\lambda = 0$) is dependent on the Prandtl number. It should be noted that in the case of nonsymmetrical internal heat liberation, the pattern of convective stability is not (as it is in [5]) invariant in relation to the shift in the direction of lateral movement of the liquid.

Figure 2 shows the family of neutral curves Ra(k) of the lower unperturbed stationary mode. The curves were plotted for $\delta = 3$ and Pr = 1 at different values of the Péclet number (Pe = 0, 1, 2, 3, and 4). The zones of instability are located above the curves. An increase in injection rate with a fixed value of the Frank-Kamenetskii parameter for the lower stationary mode reduces heating of the liquid caused by internal heat liberation (see Fig. 1) and narrows the region of unstable density stratification at the top boundary of the layer [1]. This situation leads to a substantial increase in the convective stability of the medium with an increase in the value of Pe. The marked shift in critical wave numbers k_* — corresponding to min $Ra(k) \equiv Ra_*$ — with an increase in Péclet number is a consequence of the narrowing of the zone in which convection develops.

The results of the solution to the stability problem are conveniently represented by curves describing the dependence of the minimum critical Rayleigh number Ra_* of the base level of instability on the remaining parameters. Figure 3 shows the function $Ra_*(\delta)$ for Pr = 1,and Pe = 0, 2, 4, and 6. At $\delta = \delta_{cr}$, curves $Ra_*(\delta)$ have end points. In accordance with [1], the positions of the curves are determined by the values of the Péclet numbers. The function $Ra_*(\delta_{cr})$ is shown by the dashed line in Fig. 3. (As already noted, at $\delta > \delta_{cr}$, unperturbed states cannot exist in the system, and the problem of their stability does not arise.) In the region of the existence of unperturbed states, for $Ra > Ra_*(\delta)$ the stationary modes are unstable with respect to the occurrence of convection. Intensification of injection has a stabilizing effect. The capacity of the internal chemical heat sources and, accordingly, heating of the reactant increases with an increase in the Frank-Kamenetskii parameter, leading to a reduction in convective stability. For liquid reactants, an increase in Prandtl number (for explosive liquids, Pr ~ 20) is accompanied by a slight increase in the threshold of the onset of convection. The function $Ra_*(\delta)$ corresponding to Pe = 4 and Pr = 20 is shown by the dash–dot line in Fig. 3.

The critical values of the Rayleigh number Ra_* monotonically increase with an increase in reactant injection rate, while at all values of the Frank-Kamenetskii parameter the increase in convective stability proves to be quite substantial. For example, in a layer of reactant 1 cm thick having properties similar to water, lateral movement at a rate of 0.005 cm/sec increases stability fourfold.

The dependence of the boundary of a convective stability on the Prandtl number $Ra_*(Pr)$ (plotted in Fig. 4 at $\delta = 3$ and Pe = 3.75, 4.00, and 4.25) is fairly complex in appearance. In the region Pr > 1, as noted above, the stability of the system increases by an insignificant amount. At large values of Pr, curves $Ra_*(Pr)$ are extended on an asymptote corresponding to

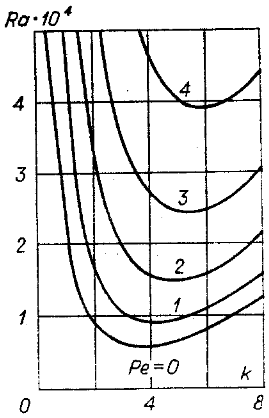

Fig. 2

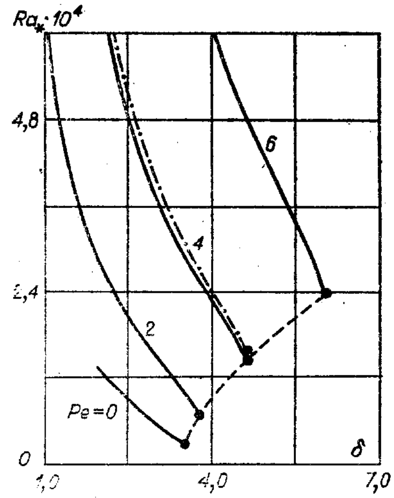

Fig. 3

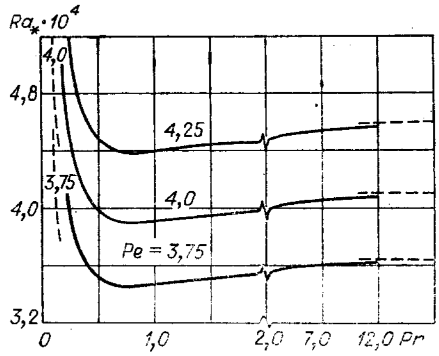

Fig. 4

Pr = ∞ (dashed straight lines in Fig. 4). The value of Ra* at Pr = ∞ exceeds the value of Ra* at Pr = 1 by only 4-5%. Most chemically active liquids are characterized by high values of Pr, and in solving boundary-value problem (4), (5) we may ignore the term in the equation of motion describing injection of the reactant. In this asymptotic case, Ra_* becomes independent of Pr.

At Prandtl numbers Pr ~ 1, the functions Ra*(Pr) have a minimum. For values Pr < 1 (reacting gases), there is a sharp increase in the stability threshold with a decrease in Pr. At Pr → 0, a boundary layer is formed at the upper boundary z = 1. In the case Pr = 0, Eq. (4) has a singularity, and to explain the behavior of function $Ra_*(Pr)$ at low values of Pr, use should be made of the method of combining asymptotic expansions [9]. It turns out that to determine the asymptotes in the first order with respect to Pr, it is sufficient to perform an outward expansion only. Here, the "viscous" terms of system of equations (4) are neglected and the order of boundary-value problem (4), (5) is reduced by unity. An inward expansion localized close to boundary z = 1 introduces a correction into higher-order (with respect to Pr) solutions, and its approximation may be ignored in the present discussion. Thus, for values of Pr close to zero, the stability boundary is determined from a boundary-value problem of the following form:

$$-L(w''-k^2w)+\mathrm{Pe}(w'''-k^2w')=-Ak^2\theta,$$
$$-L\theta+\mathrm{Pe}\,\theta'+w\Theta_0'=(\theta''-k^2\theta)+\delta\theta\exp\Theta_0; \tag{6}$$

$$w=0,\ w'=0,\ \theta=0 \quad \text{at} \quad z=0,$$
$$w=0,\ \theta=0 \quad \text{at} \quad z=1, \tag{7}$$

where L = λ Pr; A = Ra Pr.

The solutions to problem (6), (7) are shown by dashed curves in Fig. 4 for $\delta = 3$ and $Pe = 3.75$ and 4.00. When the Prandtl number $Pr \to 0$, for all values of the parameters the critical Rayleigh number $Ra_* \to \infty$.

The results obtained here show that lateral injection of a reactant is an effective means of influencing the convective stability of reactive systems.

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NUMERICAL STUDY OF THE UNSTABLE INTERACTION OF A SUPERSONIC STREAM WITH A FLAT BARRIER

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Numerous experimental studies have been devoted to the interaction of an axisymmetric supersonic stream and a flat obstacle. For example, in [1-8], determinations were made of the boundaries of the zone of instability [7] and the amplitude–frequency characteristics of the process, and features of the pattern of flow were described qualitatively within a fairly broad range of modes of interaction. In [9-13], different hypotheses were advanced on the mechanism of the appearance of oscillations. Also, within the framework of different models, analytical solutions were constructed and were used to determine frequency characteristics of the process or the lower boundary of the stability zone. In [14], a numerical study was made of one mode of nonstationary interaction between a supersonic stream and a barrier of finite dimensions.

The present work examines the unstable interaction of a supersonic stream with an infinite barrier. The problem was solved within the framework of a model of a nonviscous, non-heat-conducting gas in accordance with the difference scheme of Godunov. The potential of this scheme for solving several problems of unsteady gasdynamics was examined in [15]. In [16], Godunov's method was successfully used to calculate stationary modes of interaction between a supersonic stream and a flat barrier.

The calculations were performed on a uniform rectangular grid. The distance from the symmetry axis to the top boundary of the grid N was made larger than the diameter of the maximum cross section of the first roll of the free stream determined from the data in [17].

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